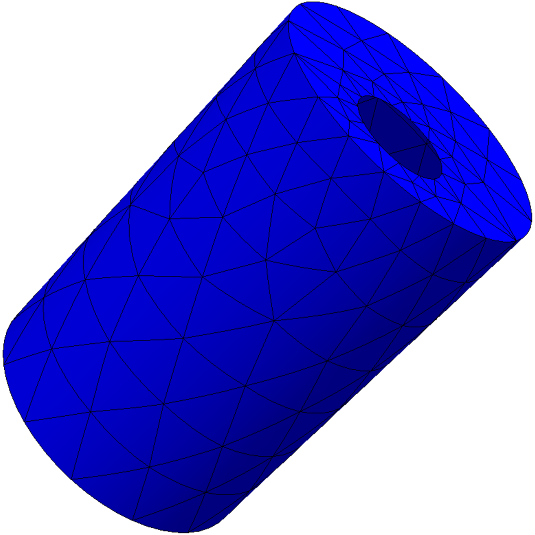
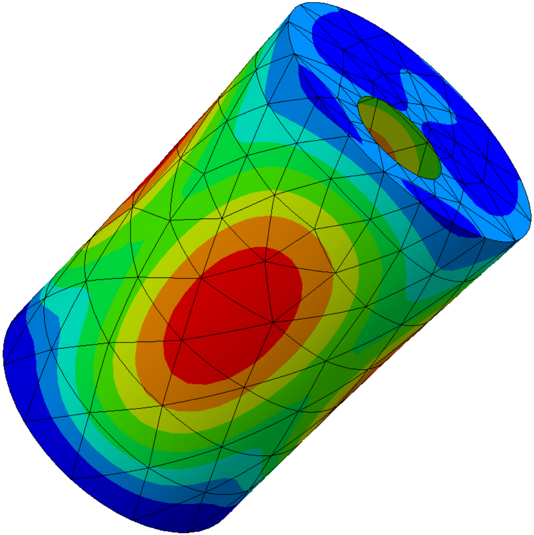
The Hellan–Herrmann–Johnson and TDNNS method for nonlinear Koiter and Naghdi shells

Michael Neunteufel (TU Wien)
Joachim Schöberl (TU Wien)

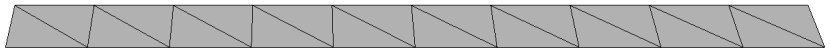
10th GACM Colloquium on Computational Mechanics, Vienna, Sep 11th, 2023

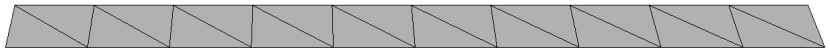
Motivation



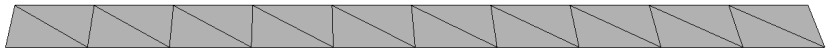


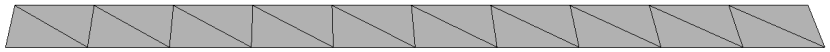
Motivation





Motivation

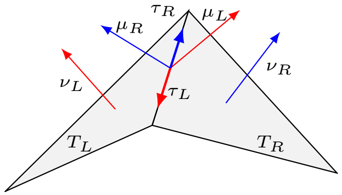




Contents

Nonlinear shells

- Normal vector ν
  Tangent vector τ
  Element normal vector $\mu = \nu \times \tau$

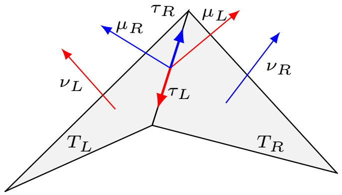


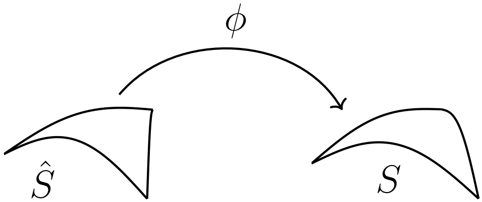
Differential geometry

- Normal vector $\hat{\nu}$

  Tangent vector $\hat{\tau}$

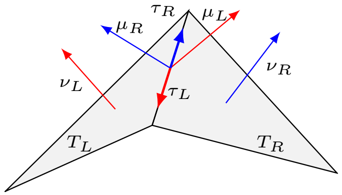
  Element normal vector $\hat{\mu} = \hat{\nu} \times \hat{\tau}$

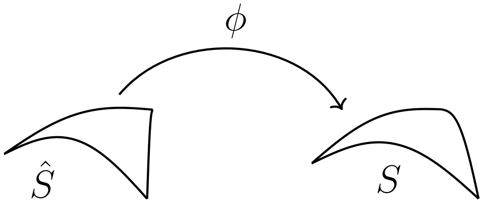




- Normal vector $\hat{\nu}$
  Tangent vector $\hat{\tau}$
  Element normal vector $\hat{\mu} = \hat{\nu} \times \hat{\tau}$

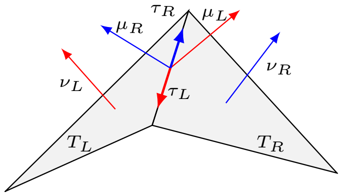
- $\boldsymbol{F} = \nabla_{\hat{\tau}} \phi$, $J = \sqrt{\det(\boldsymbol{F}^\top \boldsymbol{F})}$



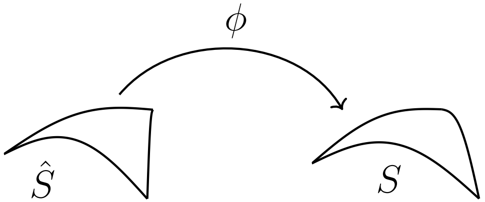


Differential geometry

- Normal vector $\hat{\nu}$
  Tangent vector $\hat{\tau}$
  Element normal vector $\hat{\mu} = \hat{\nu} \times \hat{\tau}$

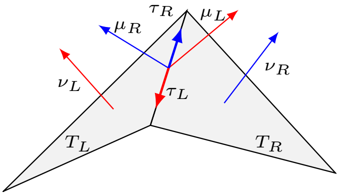


- $\boldsymbol{F} = \nabla_{\hat{\tau}}\phi$, $J = \|\operatorname{cof}(\boldsymbol{F})\|_F$

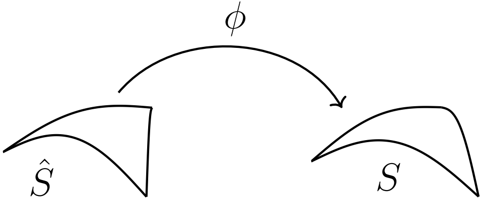


Differential geometry

- Normal vector $\hat{\nu}$
  Tangent vector $\hat{\tau}$
  Element normal vector $\hat{\mu} = \hat{\nu} \times \hat{\tau}$



- $\boldsymbol{F} = \nabla_{\hat{\tau}} \phi$, $J = \|\operatorname{cof}(\boldsymbol{F})\|_F$
- $\nu \circ \phi = \frac{1}{J} \operatorname{cof}(\boldsymbol{F}) \hat{\nu}$
  $\tau \circ \phi = \frac{1}{J_B} \boldsymbol{F} \hat{\tau}$
  $\mu \circ \phi = \nu \circ \phi \times \tau \circ \phi$



Koiter shell

$$\mathcal{W}(u) = \frac{t}{2}\|\boldsymbol{E}(u)\|_{\mathbb{M}}^2 + \frac{t^3}{24}\|\boldsymbol{F}^T\nabla(\nu\circ\phi) - \nabla\hat{\nu}\|_{\mathbb{M}}^2$$

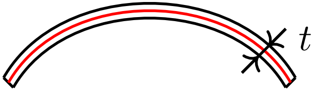
u . . . displacement of mid-surface

t . . . thickness

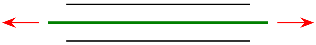
$\mathbb{M}$ . . . material tensor

$\boldsymbol{F} = \nabla u + \boldsymbol{P} = \nabla\phi, \qquad \boldsymbol{P} = \boldsymbol{I} - \hat{\nu}\otimes\hat{\nu}$

$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^\top\boldsymbol{F} - \boldsymbol{P}) = \frac{1}{2}(\nabla u^\top\nabla u + \nabla u^\top\boldsymbol{P} + \boldsymbol{P}\nabla u)$



$$\mathcal{W}(u) = \frac{t}{2}\|\boldsymbol{E}(u)\|^2_{\mathbb{M}} + \frac{t^3}{24}\|\boldsymbol{F}^T\nabla(\nu\circ\phi) - \nabla\hat{\nu}\|^2_{\mathbb{M}}$$



membrane energy

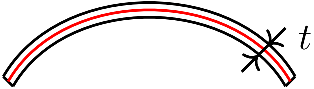
$u\ldots$ displacement of mid-surface

$t\ldots$ thickness

$\mathbb{M}\ldots$ material tensor

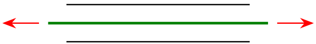
$\boldsymbol{F} = \nabla u + \boldsymbol{P} = \nabla\phi, \qquad \boldsymbol{P} = \boldsymbol{I} - \hat{\nu}\otimes\hat{\nu}$

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^\top\boldsymbol{F} - \boldsymbol{P}) = \frac{1}{2}(\nabla u^\top\nabla u + \nabla u^\top\boldsymbol{P} + \boldsymbol{P}\nabla u)$$



Koiter shell

$$\mathcal{W}(u) = \frac{t}{2}\|\boldsymbol{E}(u)\|^2_{\mathbb{M}} + \frac{t^3}{24}\|\boldsymbol{F}^T\nabla(\nu\circ\phi) - \nabla\hat{\nu}\|^2_{\mathbb{M}}$$



membrane energy

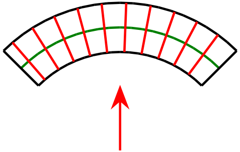
u ... displacement of mid-surface

t ... thickness

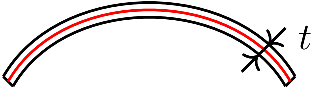
$\mathbb{M}$ ... material tensor

$\boldsymbol{F} = \nabla u + \boldsymbol{P} = \nabla\phi, \qquad \boldsymbol{P} = \boldsymbol{I} - \hat{\nu}\otimes\hat{\nu}$

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^\top\boldsymbol{F} - \boldsymbol{P}) = \frac{1}{2}(\nabla u^\top\nabla u + \nabla u^\top\boldsymbol{P} + \boldsymbol{P}\nabla u)$$



bending energy



Naghdi shell

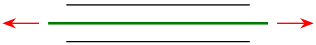
$$\mathcal{W}(u,\gamma) = \frac{t}{2}\|\boldsymbol{E}(u)\|^2_{\mathbb{M}} + \frac{t^3}{24}\|\mathrm{sym}(\boldsymbol{F}^T\nabla(\tilde{\nu}\circ\phi)) - \nabla\hat{\nu}\|^2_{\mathbb{M}} + \frac{t\kappa G}{2}\|\boldsymbol{F}^T\tilde{\nu}\circ\phi\|^2$$

γ ... shearing

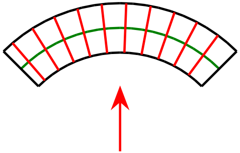
$\tilde{\nu} = \dfrac{\nu+\gamma}{\|\nu+\gamma\|}$ ... director

G ... shearing modulus

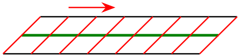
$\kappa = 5/6$ ... shear correction factor



membrane energy



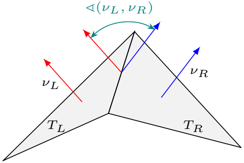
bending energy



shearing energy

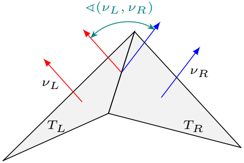
Distributional curvature for Koiter shell

Lifting: $\displaystyle\int_{\mathscr{T}_h} \boldsymbol{\kappa} : \boldsymbol{\sigma}\, dx = \sum_{T\in\mathscr{T}_h} \int_T \nabla\nu : \boldsymbol{\sigma}\, dx + \sum_{E\in\mathscr{E}_h} \int_E \sphericalangle(\nu_L,\nu_R)\boldsymbol{\sigma}_{\mu\mu}\, ds$



N., Schöberl, Sturm, Numerical shape optimization of Canham-Helfrich-Evans bending energy, *J. Comput. Phys.* (2023).

Distributional curvature for Koiter shell



Lifting: $\displaystyle\int_{\mathcal{T}_h} \boldsymbol{\kappa} : \boldsymbol{\sigma}\, dx = \sum_{T\in\mathcal{T}_h}\int_T \nabla\nu : \boldsymbol{\sigma}\, dx + \sum_{E\in\mathcal{E}_h}\int_E \sphericalangle(\nu_L,\nu_R)\boldsymbol{\sigma}_{\mu\mu}\, ds$

- Lifted curvature difference $\boldsymbol{\kappa}^{\mathrm{diff}}$ via three-field Hu–Washizu formulation

$$\mathcal{L}(u,\boldsymbol{\kappa}^{\mathrm{diff}},\boldsymbol{\sigma}) = \frac{t}{2}\|\boldsymbol{E}(u)\|^2_{\mathbb{M}} + \frac{t^3}{12}\|\boldsymbol{\kappa}^{\mathrm{diff}}\|^2_{\mathbb{M}} - \langle f,u\rangle + \sum_{T\in\mathcal{T}_h}\int_T \left(\boldsymbol{\kappa}^{\mathrm{diff}} - (\boldsymbol{F}^T\nabla(\nu\circ\phi) - \nabla\hat{\nu})\right) : \boldsymbol{\sigma}\, dx$$
$$+ \sum_{E\in\mathcal{E}_h}\int_E (\sphericalangle(\nu_L,\nu_R) - \sphericalangle(\hat{\nu}_L,\hat{\nu}_R))\boldsymbol{\sigma}_{\hat{\mu}\hat{\mu}}\, ds$$

- Lagrange parameter $\boldsymbol{\sigma}\in M_h^{k-1}$ moment tensor
- Eliminate $\boldsymbol{\kappa}^{\mathrm{diff}}$ $\rightarrow$ two-field formulation in $(u,\boldsymbol{\sigma})$

N., Schöberl: The Hellan–Herrmann–Johnson and TDNNS method for linear and nonlinear shells, *arXiv:2304.13806*.

Formulation: Koiter shell

Shell problem

Find $u \in [\mathcal{L}_h^k(\mathscr{T}_h)]^3$ and $\boldsymbol{\sigma} \in M_h^{k-1}$ for $(\boldsymbol{H}_\nu := \sum_i (\nabla^2 u_i)\nu_i)$

$$
\begin{aligned}
\mathcal{L}(u,\boldsymbol{\sigma}) =& \frac{t}{2}\|\boldsymbol{E}(u)\|^2_{\mathbb{M}} - \frac{6}{t^3}\|\boldsymbol{\sigma}\|^2_{\mathbb{M}^{-1}} - \langle f, u\rangle \\
&+ \sum_{T\in\mathscr{T}_h}\int_T \boldsymbol{\sigma} : (\boldsymbol{H}_\nu + (1-\hat{\nu}\cdot\nu)\nabla\hat{\nu})\,dx \\
&+ \sum_{E\in\mathscr{E}_h}\int_E (\sphericalangle(\nu_L,\nu_R) - \sphericalangle(\hat{\nu}_L,\hat{\nu}_R))\boldsymbol{\sigma}_{\hat{\mu}\hat{\mu}}\,ds
\end{aligned}
$$

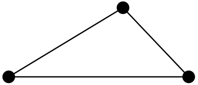
Use hybridization to eliminate $\boldsymbol{\sigma}$ $\rightarrow$ recover minimization problem

N., Schöberl: The Hellan–Herrmann–Johnson method for nonlinear shells, *Comput. Struct. 225 (2019).*

Hellan–Herrmann–Johnson finite elements

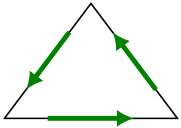
$$H^1(\Omega) = \{u \in L^2(\Omega) \,|\, \nabla u \in [L^2(\Omega)]^d\}$$

$$\mathcal{L}_h^k(\mathscr{T}_h) = \mathcal{P}^k(\mathscr{T}_h) \cap C(\Omega)$$



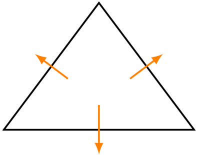
$$H(\mathrm{curl}, \Omega) = \{\sigma \in [L^2(\Omega)]^d \,|\, \mathrm{curl}\sigma \in [L^2(\Omega)]^{2d-3}\}$$

$$\mathcal{N}_{II}^k = \{\sigma \in [\mathcal{P}^k(\mathscr{T}_h)]^d \,|\, [\![\sigma_\tau]\!]_F = 0\}$$



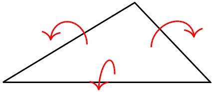
$$H(\mathrm{div}, \Omega) = \{\sigma \in [L^2(\Omega)]^d \,|\, \mathrm{div}\sigma \in L^2(\Omega)\}$$

$$BDM^k = \{\sigma \in [\mathcal{P}^k(\mathscr{T}_h)]^d \,|\, [\![\sigma_n]\!]_F = 0\}$$



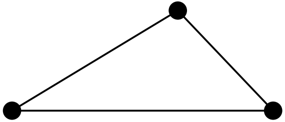
$$H(\mathrm{div}\mathbf{div}, \Omega) = \{\sigma \in [L^2(\Omega)]^{d\times d}_{\mathrm{sym}} \,|\, \mathrm{div}\mathbf{div}\sigma \in H^{-1}(\Omega)\}$$

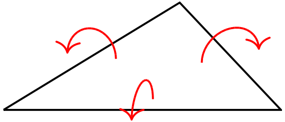
$$M_h^k(\mathscr{T}_h) = \{\sigma \in [\mathcal{P}^k(\mathscr{T}_h)]^{d\times d}_{\mathrm{sym}} \,|\, [\![n^T \sigma n]\!]_F = 0\}$$

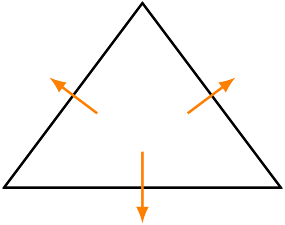


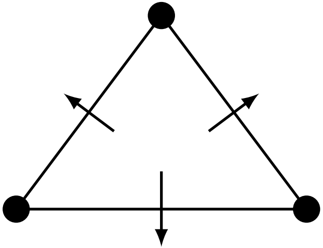
A. Pechstein and J. Schöberl: The TDNNS method for Reissner-Mindlin plates, *J. Numer. Math. (2017) 137, pp. 713-740.*

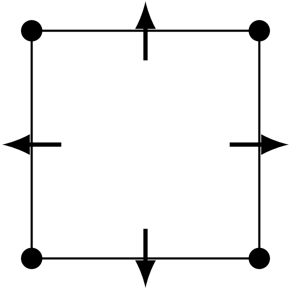
Shell element (Koiter)





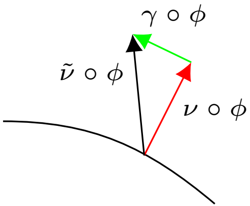






Extension to nonlinear Naghdi shells

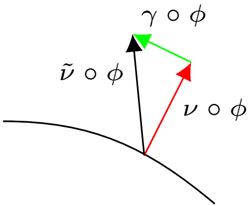
- Use hierarchical shell model
- Additional shearing dofs γ in $H(\mathrm{curl})$
- $\tilde{\nu} \circ \phi = \frac{\nu\circ\phi+\gamma\circ\phi}{\|\nu\circ\phi+\gamma\circ\phi\|}$
- Free of shear locking



Echter, R. and Oesterle, B. and Bischoff, M.: A hierarchic family of isogeometric shell finite elements, *Comput. Methods Appl. Mech. Engrg* (2013) 254, pp. 170–180.

Extension to nonlinear Naghdi shells

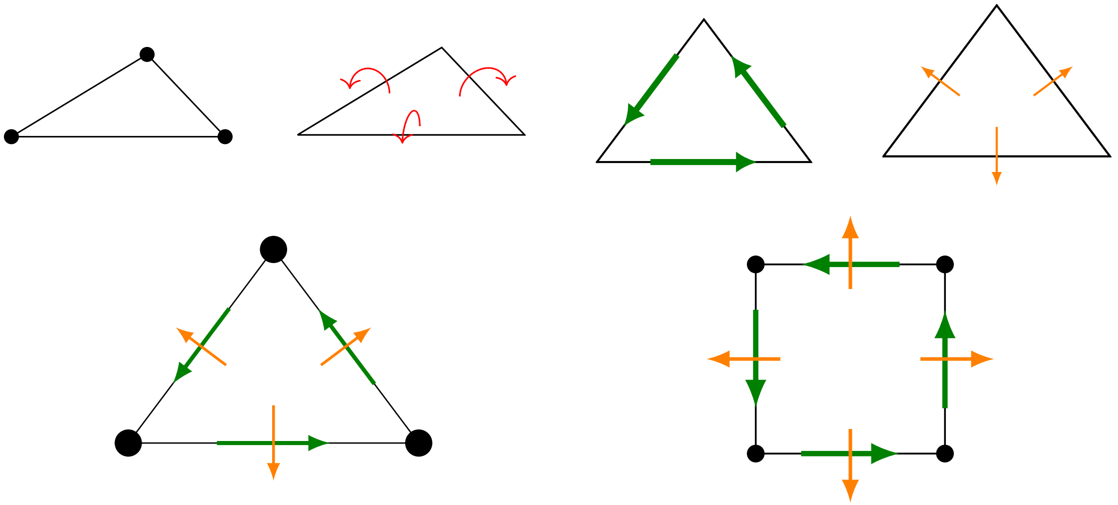
- Use hierarchical shell model
- Additional shearing dofs γ in $H(\mathrm{curl})$
- $\tilde{\nu}\circ\phi = \nu\circ\phi + \gamma\circ\phi = \frac{1}{J}\,\mathrm{cof}(\boldsymbol{F})\hat{\nu} + (\boldsymbol{F}^\dagger)^\top\hat{\gamma}$
- Free of shear locking



$$
\begin{aligned}
\mathcal{L}(u,\boldsymbol{\sigma},\hat{\gamma}) = {} & \frac{t}{2}\|\boldsymbol{E}(u)\|^2_{\mathbb{M}} + \frac{t\kappa G}{2}\|\hat{\gamma}\|^2 - \frac{6}{t^3}\|\boldsymbol{\sigma}\|^2_{\mathbb{M}^{-1}}\\
& + \sum_{T\in\mathscr{T}_h}\int_T (\boldsymbol{H}_{\tilde{\nu}} + (1-\tilde{\nu}\cdot\hat{\nu})\,\nabla\hat{\nu} - \nabla\hat{\gamma}) : \boldsymbol{\sigma}\,dx\\
& + \sum_{E\in\mathscr{E}_h}\int_E (\sphericalangle(\nu_L,\nu_R) - \sphericalangle(\hat{\nu}_L,\hat{\nu}_R) + [\![\hat{\gamma}_{\hat{\mu}}]\!])\,\boldsymbol{\sigma}_{\hat{\mu}\hat{\mu}}\,ds
\end{aligned}
$$

N., Schöberl: The Hellan–Herrmann–Johnson and TDNNS method for linear and nonlinear shells, *arXiv:2304.13806*.

Shell element (Naghdi)

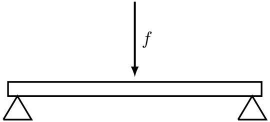


Linear Kirchhoff–Love shell and plate

$$\mathcal{L}_{\mathrm{lin}}^{\mathrm{shell}}(u,\boldsymbol{\sigma}) = \frac{t}{2}\|\mathrm{sym}(\nabla^{\mathrm{cov}} u)\|^2_{\mathbb{M}} - \frac{6}{t^3}\|\boldsymbol{\sigma}\|^2_{\mathbb{M}^{-1}} + \sum_{T\in\mathscr{T}_h}\Big(\int_T \boldsymbol{H}_{\hat\nu} : \boldsymbol{\sigma}\,dx - \int_{\partial T}(\nabla u^\top \hat\nu)_{\hat\mu}\boldsymbol{\sigma}_{\hat\mu\hat\mu}\,ds\Big)$$

$$\mathcal{L}_{\mathrm{lin}}^{\mathrm{plate}}(w,\boldsymbol{\sigma}) = -\frac{6}{t^3}\|\boldsymbol{\sigma}\|^2_{\mathbb{M}^{-1}} + \sum_{T\in\mathscr{T}_h}\Big(\int_T \nabla^2 w : \boldsymbol{\sigma}\,dx - \int_{\partial T}\frac{\partial w}{\partial\hat\mu}\boldsymbol{\sigma}_{\hat\mu\hat\mu}\,ds\Big)$$

$$\mathbf{divdiv}\nabla^2 w = f \Leftrightarrow \begin{cases}\boldsymbol{\sigma} = \nabla^2 w,\\ \mathbf{divdiv}\boldsymbol{\sigma} = f,\end{cases}$$



M. Comodi: The Hellan-Herrmann-Johnson method: some new error estimates and postprocessing, *Math. Comp. 52 (1989) pp. 17–29.*

Linear Reissner–Mindlin shell and plate

$$\mathcal{L}_{\text{lin}}^{\text{shell}}(u,\boldsymbol{\sigma},\hat{\gamma}) = \frac{t}{2}\|\mathrm{sym}(\nabla^{\mathrm{cov}} u)\|_{\mathbb{M}}^2 + \frac{t\kappa G}{2}\|\hat{\gamma}\|^2 - \frac{6}{t^3}\|\boldsymbol{\sigma}\|_{\mathbb{M}^{-1}}^2$$
$$+ \sum_{T\in\mathcal{T}_h}\Big(\int_T (\boldsymbol{H}_{\hat{\nu}} - \nabla\hat{\gamma}) : \boldsymbol{\sigma}\,dx - \int_{\partial T}((\nabla u^\top\hat{\nu})_{\hat{\mu}} - \hat{\gamma}_{\hat{\mu}})\boldsymbol{\sigma}_{\hat{\mu}\hat{\mu}}\,ds\Big)$$

$$\mathcal{L}_{\text{lin}}^{\text{plate}}(w,\boldsymbol{\sigma},\hat{\gamma}) = \frac{t\kappa G}{2}\|\hat{\gamma}\|^2 - \frac{6}{t^3}\|\boldsymbol{\sigma}\|_{\mathbb{M}^{-1}}^2$$
$$+ \sum_{T\in\mathcal{T}_h}\Big(\int_T (\nabla^2 w - \nabla\hat{\gamma}) : \boldsymbol{\sigma}\,dx - \int_{\partial T}(\frac{\partial w}{\partial\hat{\mu}} - \hat{\gamma}_{\hat{\mu}})\boldsymbol{\sigma}_{\hat{\mu}\hat{\mu}}\,ds\Big)$$

A. Pechstein and J. Schöberl: The TDNNS method for Reissner-Mindlin plates, *J. Numer. Math.* (2017) 137, pp. 713–740.

Membrane locking

Membrane locking

$$\mathcal{W}(u) = t\, E_{\text{mem}}(u) + t^3\, E_{\text{bend}}(u) - f \cdot u, \qquad f = t^3 \tilde{f}$$

Membrane locking

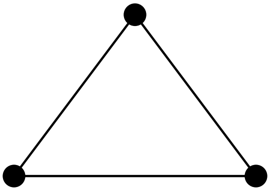
$$\mathcal{W}(u) = t^{-2} E_{\text{mem}}(u) + E_{\text{bend}}(u) - \tilde{f} \cdot u, \qquad f = t^3 \tilde{f}$$

Enforces $E_{\text{mem}}(u) = 0$ in the limit $t \to 0$

Membrane locking

$$\mathcal{W}(u) = t^{-2} E_{\text{mem}}(u) + E_{\text{bend}}(u) - \tilde{f} \cdot u, \qquad f = t^3 \tilde{f}$$

Enforces $E_{\text{mem}}(u) = 0$ in the limit $t \to 0$



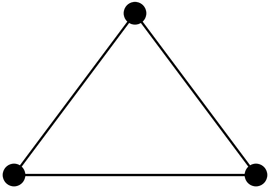
$$\mathcal{L}_h^k(\mathscr{T}_h) = \mathcal{P}^k(\mathscr{T}_h) \cap C(\Omega) \subset H^1(\Omega)$$

Membrane locking

$$\mathcal{W}(u) = t^{-2} E_{\mathrm{mem}}(u) + E_{\mathrm{bend}}(u) - \tilde{f} \cdot u, \qquad f = t^3 \tilde{f}$$

Enforces $E_{\mathrm{mem}}(u) = 0$ in the limit $t \to 0$

$$E_{\mathrm{mem}}(u) = 0 \quad \not\Rightarrow \quad E_{\mathrm{mem}}(u_h) = 0$$



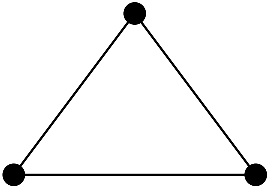
$$\mathcal{L}_h^k(\mathscr{T}_h) = \mathcal{P}^k(\mathscr{T}_h) \cap C(\Omega) \subset H^1(\Omega)$$

Membrane locking

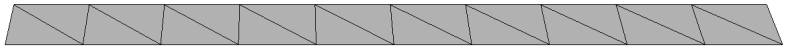
$$\mathcal{W}(u) = t^{-2} E_{\text{mem}}(u) + E_{\text{bend}}(u) - \tilde{f} \cdot u, \qquad f = t^3 \tilde{f}$$

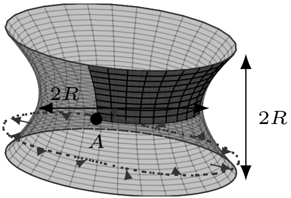
Enforces $E_{\text{mem}}(u) = 0$ in the limit $t \to 0$

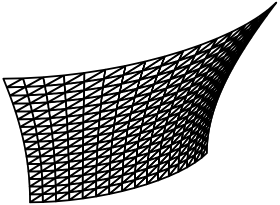
$$E_{\text{mem}}(u) = 0 \quad \not\Rightarrow \quad E_{\text{mem}}(u_h) = 0$$

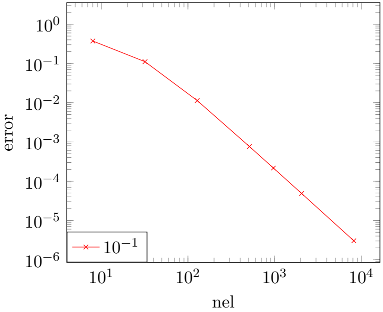


$$\mathcal{L}_h^k(\mathscr{T}_h) = \mathcal{P}^k(\mathscr{T}_h) \cap C(\Omega) \subset H^1(\Omega)$$

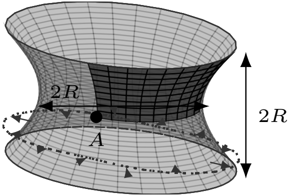


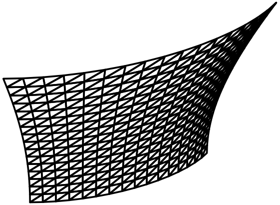

2R
2R
A

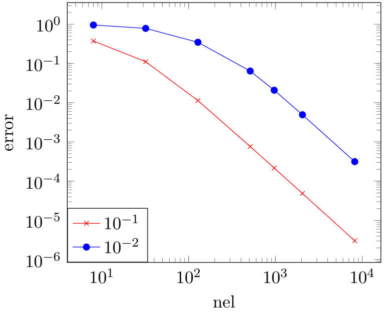



error
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{1}
10^{2}
10^{3}
10^{4}
nel
10^{-1}

Hyperboloid with free ends

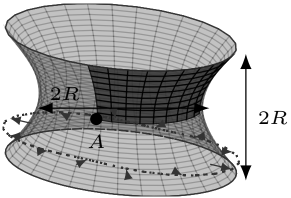


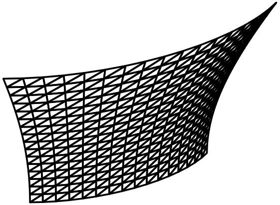


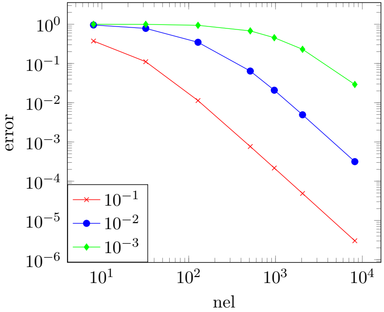


- Pre-asymptotic regime

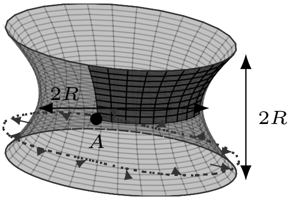
Hyperboloid with free ends

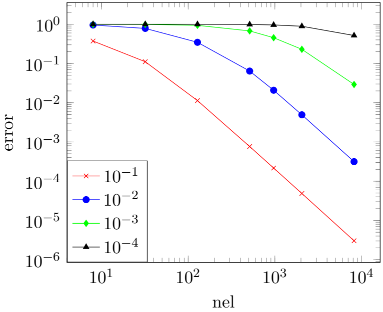


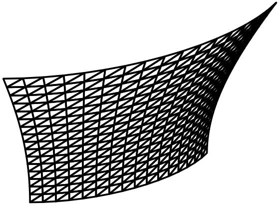




- Pre-asymptotic regime





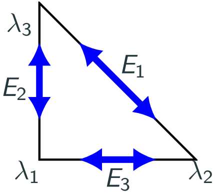


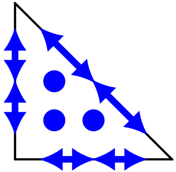
- Pre-asymptotic regime

Regge elements

$$H(\operatorname{curl}\operatorname{curl}) := \{\boldsymbol{\sigma} \in [L^2(\Omega)]^{2\times 2}_{sym} \mid \operatorname{curl}\operatorname{curl}\boldsymbol{\sigma} \in H^{-1}(\Omega)\}$$

$$\mathrm{Reg}_h^k := \{\boldsymbol{\varepsilon} \in [\mathcal{P}^k(\mathscr{T}_h)]^{d\times d}_{\mathrm{sym}} \mid [\![t^\top \boldsymbol{\varepsilon}\, t]\!]_E = 0 \text{ for all edges } E\}$$





$$\varphi_{E_i} = \nabla\lambda_j \odot \nabla\lambda_k, \qquad t_j^\top \varphi_{E_i} t_j = c_i\delta_{ij}, \qquad \varphi_{T_i} = \lambda_i\,\nabla\lambda_j \odot \nabla\lambda_k$$

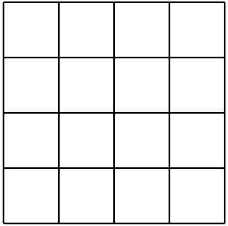
- CHRISTIANSEN: On the linearization of Regge calculus, *Numerische Mathematik 119, 4 (2011).*
- LI: Regge Finite Elements with Applications in Solid Mechanics and Relativity, *PhD thesis, University of Minnesota (2018).*
- N.: Mixed Finite Element Methods For Nonlinear Continuum Mechanics And Shells, *PhD thesis, TU Wien (2021).*

Membrane locking

$$\frac{1}{t^2}\| \quad \boldsymbol{E}(u_h)\|_{\mathbb{M}}^2$$

Membrane locking

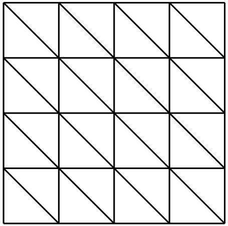
$$\frac{1}{t^2}\|\Pi^k_{L^2}\boldsymbol{E}(u_h)\|^2_{\mathbb{M}}$$

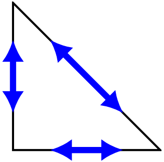


- Reduced integration for quadrilateral meshes

Membrane locking

$$\frac{1}{t^2}\|\mathcal{I}_{\mathcal{R}}^{k}\boldsymbol{E}(u_h)\|_{\mathbb{M}}^2$$

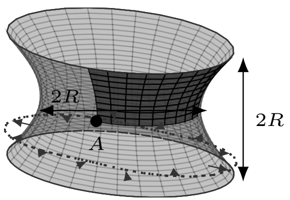


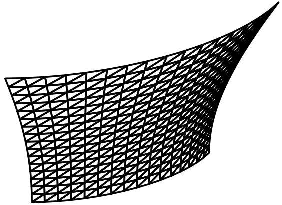


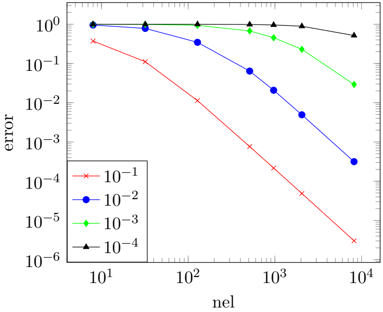
- Reduced integration for quadrilateral meshes
- Regge interpolant for triangles
- Connection to MITC shell elements

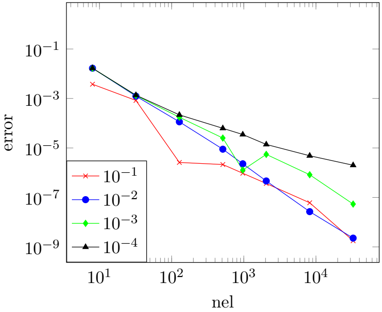
N., SCHÖBERL: Avoiding membrane locking with Regge interpolation, *Comput. Methods Appl. Mech. Engrg 373 (2021).*

Hyperboloid with free ends



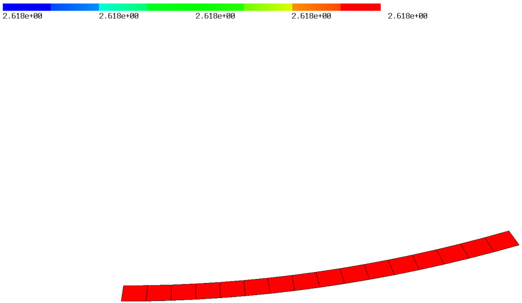


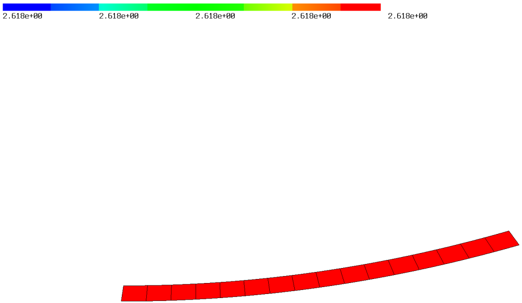




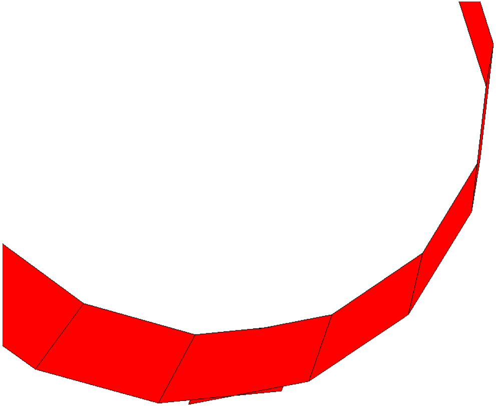
Numerical examples

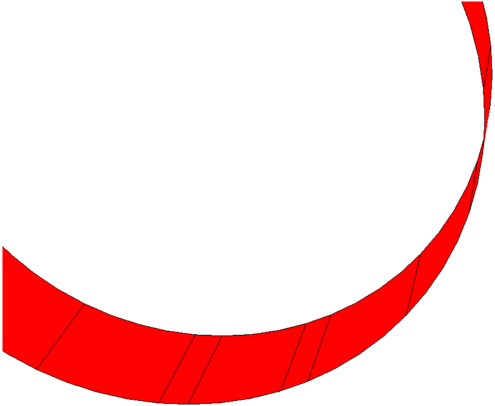
Cantilever subjected to end moment



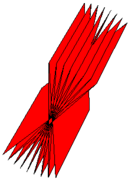


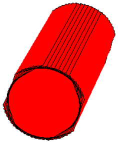
Cantilever subjected to end moment

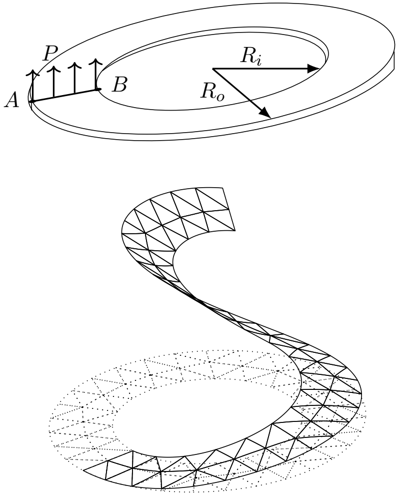


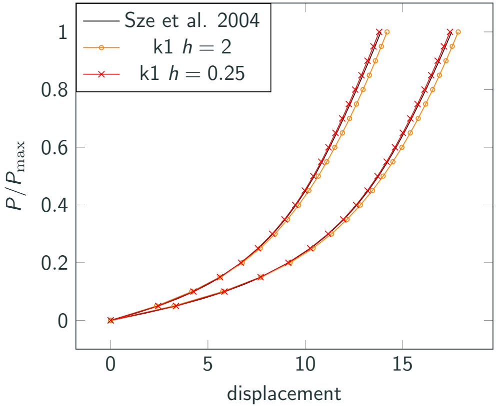


Cantilever subjected to end moment

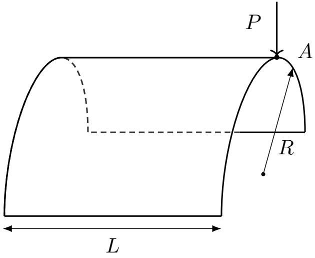


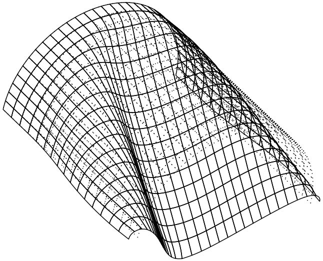


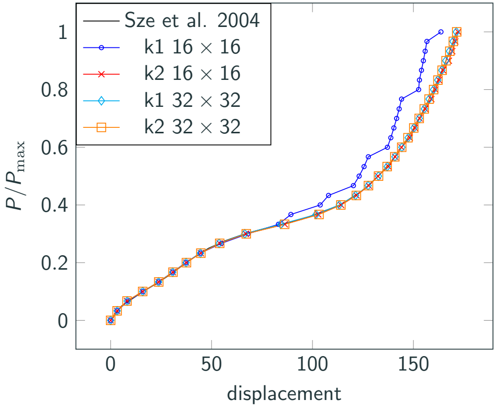

P
A
B
R_i
R_o


Sze et al. 2004
k1 $h = 2$
k1 $h = 0.25$
$P/P_{\max}$
displacement
0
0.2
0.4
0.6
0.8
1
0
5
10
15

Pinched cylinder







Summary & outlook

- Lifting of distributional curvature via Hu–Washizu three-field formulation
- HHJ for (non)linear Koiter shells
- TDNNS for (non)linear Naghdi shells
- Avoiding membrane locking with Regge elements

Summary & outlook

TECHNISCHE UNIVERSITÄT WIEN

- Lifting of distributional curvature via Hu–Washizu three-field formulation
- HHJ for (non)linear Koiter shells
- TDNNS for (non)linear Naghdi shells
- Avoiding membrane locking with Regge elements

- Coupling for 3D elasticity (A. Pechstein, M. Krommer; JKU)
- NGSolve Add-On

Literature

- N., SCHÖBERL: The Hellan–Herrmann–Johnson and TDNNS method for linear and nonlinear shells, *arXiv:2304.13806*.
- N., SCHÖBERL: The Hellan–Herrmann–Johnson method for nonlinear shells, *Comput. Struct. 225 (2019)*.
- N., SCHÖBERL: Avoiding membrane locking with Regge interpolation, *Comput. Methods Appl. Mech. Engrg 373 (2021)*.
- N.: Mixed Finite Element Methods For Nonlinear Continuum Mechanics And Shells, *PhD thesis, TU Wien (2021)*.
- N., SCHÖBERL, STURM, Numerical shape optimization of Canham-Helfrich-Evans bending energy, *J. Comput. Phys.* (2023).

Thank You for Your attention!